SYMMETRY IN PHYSICS: WIGNER'S LEGACY

The role of symmetry in physics has evolved greatly during this century. Eugene Wigner made profound contributions to this development.

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Until the twentieth century, principles of symmetry played little explicit role in theoretical physics. Conservation laws, especially those of energy and momentum, were considered to be of fundamental importance. But these were regarded as consequences of the dynamical laws of nature, rather than as consequences of the symmetries that underlay these laws. Maxwell's equations, formulated in 1865, embodied both Lorentz invariance and gauge invariance. But these symmetries of electromagnetics were not fully appreciated for 40 years or more.

Einstein's great advance in 1905 was to put symmetry first, to regard the symmetry principle as the primary feature of nature that constrains the allowable dynamical laws. Thus the transformation properties of the electromagnetic field were not to be derived from Maxwell's equations, as Hendrik Lorentz did, but rather were consequences of relativistic invariance, and indeed largely dictate the form of Maxwell's equations. This is a profound change of attitude. Lorentz must have felt that Einstein cheated.

Ten years later this point of view scored a spectacular success with Einstein's construction of general relativity. The principle of equivalence, a principle of local symmetry (the invariance of the laws of nature under local changes of the space-time coordinates), dictated the dynamics of gravity, of space-time itself.

Yet even after this magnificent success, Einstein's message had little impact on theoretical physics, except in the study of general relativity. This was partly due to the unfamiliar and rather new mathematics that was involved in exploiting symmetry principles, namely group theory. As Wigner noted in the introduction to his monumental opus, Group Theory and its Application to the Quantum Mechanics of Atomic Spectra, published in 1931, "There is great reluctance among physicists towards accepting group theoretical arguments."

Even four years later, in 1935, Edward Condon and George Shortley, in their tome on The Theory of Atomic Spectra, proudly state:

"We wish finally to make a few remarks concerning the place of the theory of groups in the study of the quantum mechanics of atomic spectra. The reader will have heard that this mathematical discipline is of great importance for the subject. We manage to get along without it. That this attitude has changed so dramatically over the last 60 years, so that today principles of symmetry are regarded as the most fundamental part of our description of nature, is in no small part due to the influence of Eugene Wigner."

Symmetry in quantum mechanics

Wigner was a member of the race of giants that reformulated the laws of nature after the quantum mechanical revolution of 1924–25. In a series of papers on atomic structure and molecular spectra, written between 1926 and 1928, Wigner laid the foundations both for the application of group theory to quantum mechanics and for the role of symmetry principles in quantum mechanics.

Wigner started from the description of quantum mechanical states as unit rays in a Hilbert space: $|\psi\rangle$, where $|\psi|^2 = 1$ and $\theta$ ranges from 0 to $2\pi$. Noting that physical predictions are given by transition probabilities, $|\langle \psi', \phi \rangle|^2$, he defined a symmetry transformation as a map

$T : \psi \rightarrow T\psi$, preserving $|\langle \psi, \phi \rangle|^2$

where $T$ is an operator in the Hilbert space. Wigner proved that $T$ is either

- linear and unitary: $(T\psi, T\phi) = (\psi, \phi)$; or
- anti-linear and anti-unitary: $(T\psi, T\phi) = (\phi, \psi)^*$. (The second possibility is required for the description of time-reversal invariance, in which the symmetry transformation interchanges initial and final states.)

In classical mechanics, symmetries of the equations of motion can be used to derive new solutions. Thus if the laws of motion are invariant under spatial rotations and $x(t)$ is a solution of the equations of motion, say an orbit of the Earth around the Sun, then $Rx(t)$, the spatially rotated $x(t)$, is also a solution. This is interesting and sometimes useful. (See figure 2a.)

Wigner understood, "In quantum theory, invariance principles permit even further reaching conclusions than in classical mechanics." In quantum mechanics there is a new and powerful twist due to the linearity of the symmetry transformation and the superposition principle. Thus if $|\psi\rangle$ is an allowed state then so is $R|\psi\rangle$, where $R$ is the operator in the Hilbert space corresponding to the symmetry transformation $R$. So far this is similar to classical mechanics. However, we can now superpose these states, that is, construct a new allowed state: $|\psi\rangle + R|\psi\rangle$. There is no classical analog for such a superposition of, say, two orbits of the earth.

As Wigner pointed out, the superposition principle means that we can construct linear combinations of states that transform simply under the symmetry transformations. Thus superimposing all states that are related by rotations, we obtain a state $|\psi\rangle = \sum R_i|\psi\rangle$ that is rotation-
EINSTEIN AND WIGNER (second from left) both considered symmetry principles to be of fundamental importance. Einstein’s theories of special relativity (1905) and general relativity (1915) are classic examples of symmetry principles constraining and even dictating dynamics. Wigner’s great contribution in the late 1920s and 1930s was to discover the fundamental role symmetry plays in quantum theories, from explaining atomic spectra to classifying types of elementary particles. Pictured here at Einstein’s 70th birthday celebration in 1949 at the Institute for Advanced Study in Princeton are (from left to right) Howard Robertson, Wigner, Herman Weyl, Kurt Gödel, I. I. Rabi, Einstein, Rudolf Ladenburg, J. Robert Oppenheimer and G. M. Clemence. FIGURE 1.

ally invariant:

$$R'(\Phi) = \sum_{R'} RR'\langle \Psi | = \sum_{R'} R'^\dagger \langle \Psi | = | \Phi \rangle$$

The state $| \Phi \rangle$ forms a singlet representation of the rotation group. (See figure 2b.)

Other superpositions of rotated states will yield other irreducible representations of the symmetry group. Irreducible representations are special: They cannot be further subdivided—any subset of states gets mixed by the symmetry group with all the other states of the representation. Furthermore any state can be written as a sum of states transforming according to irreducible representations of the symmetry group. Wigner realized that these special states can be used to classify all the states of a system possessing symmetries, and play a fundamental role in the analysis of such systems. Consequently he understood the important role that the theory of representations of continuous and discrete groups would play in quantum mechanics and he proceeded to develop the requisite tools in the cases of physical interest.

Representations of the rotation group

In studying atomic spectra Wigner needed to analyze the representations of the rotation group, SO(3), plus the discrete symmetries of parity and time reversal. In this context he proved the fundamental theorem that the operator $U(R)$ that corresponds to a rotation $R$ will be represented, in general, by a projective representation of SO(3). Thus if $R S = T$ then $U(R) U(S) = \omega(R, S) U(T)$, where the phase factor $\omega(R, S)$ = 1, is allowed because quantum mechanical states are rays in the Hilbert space. Wigner then eliminated this phase by showing that with an appropriate choice of the phases of the states one obtains a true faithful representation of the covering group of SO(3), which is SU(2) (the group of unitary unimodular two-by-two matrices). SU(2) has, in addition to the vector and tensor representations, spinor representations that are not faithful representations of the rotation group. (In particular a 2π rotation, which is equivalent to the identity matrix $\mathbf{1}$ in SO(3) is represented by $-\mathbf{1}$ in a spinor representation.) Thus Wigner provided the deep reason for spinors in nature, which had previously been introduced in an ad hoc fashion to describe fermions such as the electron. (See figure 3.)

With the tools of group theory that Wigner helped to develop, he derived many consequences and explained many features of atomic spectra. In particular he showed that many selection rules were simply the consequences of symmetry. Thus Laporte’s rule, allowing electric dipole transitions only between even and odd states, was a consequence of parity invariance. He put vector addition of angular momentum states on a firm mathematical footing (the Wigner–Eckhart Theorem). One of his most important contributions was pedagogical. He introduced a generation of physicists to the tools of group theory, which were to play such an important role in atomic, nuclear and then particle physics.

In the following years, during the 1930s and 1940s, Wigner applied and developed these tools for the emerging field of nuclear physics. Wigner was a mathematical physicist in the best sense of the term—namely he was first and foremost a physicist. Thus he did not shy from considering approximate symmetries of nature. A mathematician might have been reluctant to discuss symmetries that were not true symmetries at all. Not Wigner. He played an important role in developing the concept of isotopic spin symmetry, an approximate symmetry of the strong interactions. He later extended this symmetry to the SU(4) group that contained both isotopic spin and rotational symmetry in a nontrivial fashion. To combine internal and space-time symmetries in a simple group was
a daring move. It was imitated later by relativistic particle physicists in the 1960s, when they tried to combine the Lorentz group and internal symmetries, with much less success.

**Representations of the Poincaré group**

Toward the end of the 1930s Wigner turned his attention to time-dependent symmetries, invariance groups that included time-translation invariance. In 1939 he published the epochal paper on *The Unitary Representations of the Inhomogeneous Lorentz Group*. Also known as the Poincaré group, this group had not been seriously studied previously by either mathematicians or physicists. In his paper Wigner posed the question: What are the unitary representations of the Poincaré group and what is their physical significance? In the same paper he gave the complete answer to these questions.

Wigner showed that in relativistic quantum mechanics Poincaré transformations were represented by faithful representations of its covering group, ISL(2,C), and he gave a complete classification and an explicit construction of all the irreducible representations. The method was extremely ingenious.

The Poincaré group is not compact and its unitary representations are infinite dimensional. Except for the trivial representation (used to describe the vacuum state) each irreducible representation \( \pi(p^\mu, \lambda) \) is described by a set of internal indices \( \lambda \) and a continuously varying four-momentum \( p^\mu \) \( (\mu = 0, 1, 2, 3) \) corresponding to a particle of energy \( p^0 \) and momentum \( p^\mu \) of fixed mass \( M = \sqrt{p^0 p^0} \). Wigner found that the irreducible representations can be labeled by the eigenvalues of two Casimir operators; one being the mass and the other related to the angular momentum. Wigner showed that to construct the representation it was sufficient to consider the subgroup (the “little group”) of transformations that leave the four-momentum of the particle invariant, since every Lorentz transformation can be written as a product of boosts and elements of the little group. There are then two types of physical representations:

- Massive representations: \( M > 0 \). Here the little group is simply the rotation group, so that there is one irreducible representation for each irreducible representation of \( SU(2) \).
- Massless representations: \( M = 0 \). Here the little group is the two-dimensional Euclidean group of two translations and one rotation of the 2-plane. The only finite-dimensional representations of this group are one-dimensional, on which the translations act trivially. These are labeled by a single helicity \( \lambda \) that is a half-integer or integer. An example of such a representation is the left-handed neutrino, which only has one helicity state with \( \lambda = -\frac{1}{2} \). (If we include parity then irreducible representations contain both positive and negative helicities \( \pm \lambda \).) Wigner’s analysis made it clear that massless spinning particles are fundamentally different from massive particles. This difference has profound implications for dynamics; indeed it requires that massless spin-1 particles be described by gauge theories. (See figure 4.)

Wigner provided a complete classification of all elementary particles. Indeed his analysis provided a *definition* of what we mean by an elementary particle, which according to Wigner should be identified as an irreducible representation of the Poincaré group. Wigner took this definition very seriously. I remember with fondness many long discussions, arguments actually, with Eugene over tea at Princeton about quarks. Wigner found it very hard to accept the physical reality of quarks since, due to con-

**Symmetry operations generate new solutions from old when the underlying dynamics is symmetrical.** a: Applying a rotation operator to the Earth’s orbit around the Sun produces a different possible orbit. b: The same can be done in quantum mechanics, here to a wavefunction of a hydrogen atom. The superposition of all possible rotations of the wavefunction is itself a spherically symmetric wavefunction, in the singlet representation of the rotation group. From this Wigner saw the profound importance of irreducible representations in quantum mechanics. **Figure 2.**
The rotation group: Because quantum mechanical states are rays in a Hilbert space, Wigner was led from the group of rotations SO(3) to its covering group, SU(2). The irreducible representations of SU(2) then correspond to types of particles. In particular spinors, required to describe electrons, quarks and neutrinos, arise naturally. Figure 3.

Reflections on symmetry

For his work on the “discovery and application of fundamental symmetry principles” Wigner received the Nobel Prize in 1963. In his Nobel lecture and elsewhere he presented a sweeping analysis of the role of symmetry principles in nature. He pointed out that progress in physics was partly based on the ability to separate the analysis of a physical phenomenon into two parts. First there are the initial conditions that are arbitrary, complicated and unpredictable. Then there are the laws of nature that summarize the regularities that are independent of the initial conditions. Symmetry principles play a similar role with respect to the laws of nature. They summarize the regularities of the laws that are independent of the specific dynamics. Thus Wigner argued that invariance principles provide a structure and coherence to the laws of nature just as the laws of nature provide a structure and coherence to a set of events. (See figure 5.)

Indeed it is hard to imagine that much progress could have been made in deducing the laws of nature without the existence of certain symmetries. Thus the ability to repeat experiments at different places and at different times is based on the invariance of the laws of nature under spacetime translations. Without regularities embodied in the laws of physics we would be unable to make sense of physical events; without regularities in the laws of nature we would be unable to discover the laws themselves.

The Poincaré group corresponds to the symmetries of special relativity—Lorentz transformations and translations in space and time. Wigner provided a complete classification of all elementary particles by analyzing the irreducible representations of the covering group, ISL(2, C). Figure 4.

Symmetry after Wigner: Gauge invariance

Wigner excluded gauge symmetries from his discussion of fundamental symmetries. He recognized that these were of a totally different nature. According to Wigner gauge invariance was a dynamical symmetry, in contrast to Lorentz invariance, which is a geometrical symmetry. Traditional symmetries are regularities of the laws of motion but are formulated in terms of physical events; the application of the symmetry transformation yields a different physical situation. On the other hand gauge symmetries are formulated only in terms of the laws of nature; the application of the symmetry transformation merely changes our description of the same physical situation. (See figure 6.) Consequently Wigner regarded these invariance principles as rather artificial. In the case of electrodynamics he remarked

This gauge invariance is, of course, an artificial one, similar to that which we could obtain by introducing into our equations the location of a ghost. The equations must then be invariant with respect to changes of coordinates of that ghost. One does not see, in fact, what good the introduction of the coordinate of the ghost does.

This attitude toward gauge invariance has changed dramatically in the last two decades. Gauge theories have assumed a central position in the fundamental theories of nature. They provide the basis for the extremely successful standard model, a theory of the fundamental, nongravitational forces of nature—the electromagnetic, weak and strong interactions. To be sure, gauge invariance
Broken symmetry and new symmetries

The secret of nature is symmetry but much of the texture of the world is due to mechanisms of symmetry breaking. The spontaneous symmetry breaking of global and local gauge symmetries is a recurrent theme in modern theoretical physics. This phenomena is special to theories with an infinite number of degrees of freedom, in which global symmetries may be realized in two different ways. The first way (sometimes referred to as the Wigner–Weyl mode) is standard: The laws of physics are invariant and the ground state of the theory, the vacuum, is unique and symmetric. This is always the case for quantum mechanical systems with a finite number of degrees of freedom; the kind of systems that Wigner mostly analyzed. In systems with an infinite number of degrees of freedom, however, a second mode (sometimes called the Nambu–Goldstone mode) is possible, in which the ground state is asymmetric. Such spontaneous symmetry breaking is responsible for magnetism, superconductivity, the structure of the unified electroweak theory and more. Indeed the search for new symmetries of nature is based on this possibility, for a new symmetry that we discover must be somehow broken, otherwise it would have been apparent long ago—it would have been an old symmetry.

Current theoretical exploration in the search for further unification of the forces of nature, including gravity, is largely based on the search for new symmetries of nature. Theorists speculate on larger and larger local symmetries and more intricate patterns of symmetry breaking in order to further unify the separate interactions. Most exciting is the speculation concerning new kinds of symmetry, which could explain some of the most mysterious features of nature. Foremost among these is supersymmetry, a profound and beautiful extension of the geometric symmetries of space-time to include symmetries generated by fermionic (anticommuting) charges. An ordinary symmetry, such as that of the electroweak force, is generated by bosonic (commuting) charges; it groups bosons together (the force-carrying W and Z boson and the photon, for example) and groups fermions together (the electron and electron neutrino). Supersymmetric theories, however, have the ability to unify bosons and fermions into a single pattern, to unify matter and force and to help explain the mysterious fact—the hierarchy problem—that the mass scale of atomic and nuclear physics is so much smaller than the scale determined by gravity (the Planck mass of about 10^{19} GeV). We eagerly await the experimental discovery of the signs of (spontaneously broken) supersymmetry at the next generation of particle accelerators.

Finally, in recent years we have begun to seriously explore a new kind of theory based on a radical extension of the conceptual framework of local quantum field theory—string theory. String theory is the most ambitious candidate for a unified theory of all the interactions that naturally embodies quantum gravity in a consistent fashion. It contains within it all the familiar symmetries that we have discovered play a role in nature. In addition, there are hints within the theory that it embodies strange new symmetries that we are now trying to understand.

The recent progress in constructing the standard model and in going beyond it has certainly justified Wigner’s point of view regarding the fundamental role of symmetry in physics. If we have gone so far in our understanding of symmetry in this century, it is in large part by standing on the shoulders of giants such as Eugene P. Wigner.

References