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Addendum

Four easy routes to the Lorentz transformations: addendum to 'Lorentz transformations and the wave equation'

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Abstract

In this paper I briefly discuss and compare four easy derivations of the Lorentz transformations. Two of these derivations assume the invariance of the Minkowski spacetime interval in inertial frames and the other two assume the invariance of the d'Alembert operator in these frames. These derivations are suitable for a first view of special relativity. Finally, I discuss the comment made by Di Rocco on my original paper, 'Lorentz transformations and the wave equation' (2016 *Eur. J. Phys.* **37** 025603).

1. Introduction

There are so many derivations of the Lorentz transformations reported in the literature that an interesting task for an instructor is to investigate which of them are appropriate to be presented in an undergraduate physics course. In a recent note [1], I have suggested a simple derivation of these transformations, which uses the standard configuration[1] and assumes the invariance of the d'Alembert operator: $\partial^2/\partial x^2 - (1/c^2)\partial^2/\partial t^2 = \partial^2/\partial x'^2 - (1/c^2)\partial^2/\partial t'^2$, which expresses the two postulates of special relativity. This derivation of the Lorentz transformation is suitable for a first view of the theory of relativity. Di Rocco [2] has commented on this derivation.

In order to put the derivation of the Lorentz transformations from the invariance of the d'Alambert operator in a pedagogical context, it is worthwhile to compare it with the usual derivation of these transformations, which uses the standard configuration and assumes the invariance of the Minkowski space-time interval: $x'^2 - c^2t'^2 = x^2 - c^2t^2$. In this paper I

[1] In the standard configuration two inertial frames S and S' are in relative motion with the speed v along their common xx' direction. The origins of the two frames coincide at the instant $t = t' = 0$. The coordinates transverse to the relative motion of the frames S and S' are assumed to be invariant: $y' = y$ and $z' = z$.

briefly review and compare four simple derivations of the Lorentz transformations formulated in the standard configuration. Two of these derivations assume the invariance of the Minkowski spacetime interval and the other two the invariance of the d'Alembert operator. I then discuss the comments made by Di Rocco [2].

2. Four easy derivations of the Lorentz transformations

Consider the following four derivations of the Lorentz transformations.

- Derivation I. The starting point is the invariance of the space-time interval:

$$x'^2 - c^2t'^2 = x^2 - c^2t^2. \tag{1}$$

The relations that transform the coordinates (x, t) into the coordinates (x', t') are linear:

$$x' = \mathrm{A}x + \mathrm{B}t, \quad t' = \mathrm{C}x + \mathrm{D}t, \tag{2}$$

where A, B, C, D are constants to be determined. The linearity of the relations in (2) is a consequence of the homogeneity of the space and time [3]. The origin of the primed frame $x' = 0$ is a point described in the unprimed frame by $x = vt$ (in agreement with the Galilean transformation $x' = x - vt$). Substituting these values in the first relation of (2) one obtains $\mathrm{B} = -v\mathrm{A}$. Therefore, the transformations in (2) take the convenient form

$$x' = \mathrm{A}(x - vt), \quad t' = \mathrm{C}x + \mathrm{D}t. \tag{3}$$

Using these transformations in (1) it becomes

$$(\mathrm{A}^2 - c^2\mathrm{C}^2)x^2 - c^2(\mathrm{D}^2 - \mathrm{A}^2v^2/c^2)t^2 - 2(\mathrm{A}^2v + c^2\mathrm{DC})xt = x^2 - c^2t^2, \tag{4}$$

which implies the system of algebraic equations

$$\mathrm{A}^2 - c^2\mathrm{C}^2 = 1,\ c^2\mathrm{D}^2 - \mathrm{A}^2v^2 = c^2,\ \mathrm{A}^2v + c^2\mathrm{DC} = 0. \tag{5}$$

This system can be solved, obtaining

$$\mathrm{A} = \mathrm{D} = \frac{1}{\sqrt{1 - v^2/c^2}}, \quad \mathrm{C} = -\frac{v}{c^2\sqrt{1 - v^2/c^2}}. \tag{6}$$

When these constants are inserted in (3) we finally obtain the Lorentz transformations:

$$x' = \gamma(x - vt), \quad t' = \gamma\left(t - \frac{vx}{c^2}\right), \tag{7}$$

where $\gamma = 1\Big/\sqrt{1 - v^2/c^2}$.

- Derivation II. The starting point is again (1) but factored as

$$(x' - ct')(x' + ct') = (x - ct)(x + ct). \tag{8}$$

This relation is identically satisfied if one writes

$$x' - ct' = A(x - ct), \quad x' + ct' = \mathrm{A}^{-1}(x + ct), \tag{9}$$

where A is independent of x and t but can depend on v and c. The relations in (9) are also linear because of the homogeneity and isotropy of the space [4]. Again, the origin of the primed frame $x' = 0$ is the point $x = vt$ described in the unprimed frame (in agreement with the Galilean transformation $x' = x - vt$). Using these conditions in (9) it follows that

$$-ct' = A(vt - ct), \quad ct' = A^{-1}(vt + ct), \tag{10}$$

which are combined to yield

$$A = \gamma\left(1 + \frac{v}{c}\right), \quad A^{-1} = \gamma\left(1 - \frac{v}{c}\right), \tag{11}$$

where $\gamma = 1\Big/\sqrt{1 - v^2/c^2}$. One then adds the relations given in (9), obtaining

$$2x' = A(x - ct) + A^{-1}(x + ct). \tag{12}$$

Equations (11) and (12) yield the Lorentz transformation for the space coordinate

$$x' = \gamma(x - vt). \tag{13}$$

One now subtracts the relations given in (9)

$$-2ct' = A(x - ct) - A^{-1}(x + ct). \tag{14}$$

From (11) and (14) it follows the Lorentz transformation for the time coordinate

$$t' = \gamma\left(t - \frac{vx}{c^2}\right). \tag{15}$$

- Derivation III. The starting point is the invariance of the d'Alembert operator:

$$\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = \frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}. \tag{16}$$

The involved transformations for the derivative operators are:

$$\frac{\partial}{\partial x} = \mathrm{A}\frac{\partial}{\partial x'} + \mathrm{C}\frac{\partial}{\partial t'}, \quad \frac{\partial}{\partial t} = \mathrm{B}\frac{\partial}{\partial t'} + \mathrm{D}\frac{\partial}{\partial x'}, \tag{17}$$

where A, B, C, D are constants to be found. The relations in (17) are linear because they are implied by the relations $x' = \mathrm{A}x + \mathrm{D}t$ and $t' = \mathrm{C}x + \mathrm{B}t$, which are in turn linear because of the homogeneity of the space and time [3]. The second relation in (17) must satisfy the condition of reducing to the corresponding Galilean transformation [4]: $\partial/\partial t = \partial/\partial t' - v\partial/\partial x'$ in an appropriate limit. A suitable transformation satisfying this condition is $\partial/\partial t = F(v, c)(\partial/\partial t' - v\partial/\partial x')$, where $F(v, c)$ depends on v and c so that $F(v, c) \to 1$ when $v \ll c$.[2] From this transformation it follows that if $\partial/\partial t = 0$ then $\partial/\partial t' = v\partial/\partial x'$ because $F(v, c) \neq 0$. Using these results in the second relation of (17) one gets $\mathrm{D} = -v\mathrm{B}$ and therefore (17) becomes

$$\frac{\partial}{\partial x} = \mathrm{A}\frac{\partial}{\partial x'} + \mathrm{C}\frac{\partial}{\partial t'}, \quad \frac{\partial}{\partial t} = \mathrm{B}\left(\frac{\partial}{\partial t'} - v\frac{\partial}{\partial x'}\right), \tag{18}$$

Using these transformations in (16) it becomes

[2] The transformation $\partial/\partial t = F(v, c)(\partial/\partial t' - v\partial/\partial x')$, which was used only to fix a constant, is not the unique in reducing to the corresponding Galilean transformation. A referee has correctly pointed out that the more complicated transformation $\partial/\partial t = F(v, c)(\partial/\partial t' - vG(v, c)\partial/\partial x')$ with $F(v, c) \to 1$ and $G(v, c) \to 1$ when $v \ll c$ has also this property. The use of this last transformation, however, does not lead to the Lorentz transformations. The origin of $\partial/\partial t = F(v, c)(\partial/\partial t' - v\partial/\partial x')$ and its associated transformation $\partial/\partial x = F(v, c)\partial/\partial x' + \mathrm{C}\partial/\partial t'$ can be traced to the transformations $x' = F(v, c)(x - vt)$ and $t' = \mathrm{C}x + F(v, c)t$. If these last transformations are used in the invariance $x'^2 - c^2t'^2 = x^2 - c^2t^2$ then we obtain expressions for $F(v, c)$ and C such as $F(v, c) \to 1$ and $C \to 0$ when $v \ll c$, and thus we get the Galilean transformations $x' = x - vt$ and $t' = t$.

$$(\mathrm{A}^2-\mathrm{B}^2v^2/c^2)\frac{\partial^2}{\partial x'^2}-\frac{1}{c^2}(\mathrm{B}^2-c^2\mathrm{C}^2)\frac{\partial^2}{\partial t'^2}-2(\mathrm{AC}+\mathrm{B}^2v/c^2)\times\frac{\partial^2}{\partial x'\partial t'}=\frac{\partial^2}{\partial x'^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t'^2}. \tag{19}$$

Form invariance demands the following system of equations

$$\mathrm{B}^2-c^2\mathrm{C}^2=1,\ \ c^2\mathrm{A}^2-\mathrm{B}^2v^2=c^2,\ \ \mathrm{B}^2v+c^2\mathrm{AC}=0. \tag{20}$$

This system is equivalent to that given in (5). The solution reads,

$$\mathrm{A}=\mathrm{B}=\frac{1}{\sqrt{1-v^2/c^2}},\ \ \mathrm{C}=-\frac{v}{c^2\sqrt{1-v^2/c^2}}, \tag{21}$$

Using (21) in (18) one obtains the Lorentz transformation of the derivative operators:

$$\frac{\partial}{\partial x}=\gamma\left(\frac{\partial}{\partial x'}-\frac{v}{c^2}\frac{\partial}{\partial t'}\right),\ \ \frac{\partial}{\partial t}=\gamma\left(\frac{\partial}{\partial t'}-v\frac{\partial}{\partial x'}\right), \tag{22}$$

where $\gamma=1\big/\sqrt{1-v^2/c^2}$. Clearly (22) imply transformations of the form $x'=x'(x,t)$ and $t'=t'(x,t)$. To find the explicit form of these transformations one uses (22) and obtains

$$\frac{\partial x'}{\partial x}=\gamma,\ \ \frac{\partial x'}{\partial t}=-\gamma v,\ \ \frac{\partial t'}{\partial t}=\gamma,\ \ \frac{\partial t'}{\partial x}=-\frac{\gamma v}{c^2}. \tag{23}$$

The first relation of (23) implies (i): $x'=\gamma x+f_1(t)$, where $f_1(t)$ can be found (up to a constant) by differentiating (i) with respect to t and using the second relation of (23): $\partial x'/\partial t=df_1(t)/dt=-\gamma v$. This last equality implies (ii): $f_1(t)=-\gamma vt+x_0$, where x_0 is a constant. From (i) and (ii) one gets $x'=\gamma(x-vt)+x_0$. The third relation of (23) implies (iii): $t'=\gamma t+f_2(x)$, where $f_2(x)$ can be obtained (up to a constant) from differentiating (iii) with respect to x and using the last relation of (23): $\partial t'/\partial x=df_2(x)/dx=-\gamma v/c^2$. This last equality implies (iv): $f_2(x)=-\gamma vx/c^2+t_0$, where t_0 is a constant. From (iii) and (iv) it follows that $t'=\gamma(t-vx/c^2)+t_0$. The origins of the frames S and S' coincide at $t=t'=0$ and then $x_0=0$ and $t_0=0$. In this way one obtains the Lorentz transformations:

$$x'=\gamma(x-vt),\ \ t'=\gamma\left(t-\frac{vx}{c^2}\right). \tag{24}$$

- Derivation IV. The starting point is again (16) but expressed as [1]

$$\left(\frac{\partial}{\partial x}-\frac{1}{c}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x}+\frac{1}{c}\frac{\partial}{\partial t}\right)=\left(\frac{\partial}{\partial x'}-\frac{1}{c}\frac{\partial}{\partial t'}\right)\left(\frac{\partial}{\partial x'}+\frac{1}{c}\frac{\partial}{\partial t'}\right). \tag{25}$$

This relation is identically satisfied if one writes

$$\left(\frac{\partial}{\partial x}-\frac{1}{c}\frac{\partial}{\partial t}\right)=A\left(\frac{\partial}{\partial x'}-\frac{1}{c}\frac{\partial}{\partial t'}\right),\quad \left(\frac{\partial}{\partial x}+\frac{1}{c}\frac{\partial}{\partial t}\right)=A^{-1}\left(\frac{\partial}{\partial x'}+\frac{1}{c}\frac{\partial}{\partial t'}\right), \tag{26}$$

where A is independent of the derivative operators but can depend on the speeds v and c. The relations in (26) are linear because of the homogeneity and isotropy of the space [4]. Following the same argument used in Derivation III, one concludes that $\partial/\partial t=0$ when

$\partial/\partial t' = v\partial/\partial x'$. When this result is used in the relations given in (26) they become

$$\frac{\partial}{\partial x} = A\left(\frac{\partial}{\partial x'} - \frac{v}{c}\frac{\partial}{\partial x'}\right), \quad \frac{\partial}{\partial x} = A^{-1}\left(\frac{\partial}{\partial x'} + \frac{v}{c}\frac{\partial}{\partial x'}\right). \tag{27}$$

These can be combined to yield the expressions

$$A = \gamma\left(1 + \frac{v}{c}\right), \quad A^{-1} = \gamma\left(1 - \frac{v}{c}\right), \tag{28}$$

where $\gamma = 1\big/\sqrt{1 - v^2/c^2}$. Next, one adds the relations given in (26),

$$2\frac{\partial}{\partial x} = A\left(\frac{\partial}{\partial x'} - \frac{1}{c}\frac{\partial}{\partial t'}\right) + A^{-1}\left(\frac{\partial}{\partial x'} + \frac{1}{c}\frac{\partial}{\partial t'}\right), \tag{29}$$

and uses (28) to obtain the Lorentz transformation for the space-derivative operator

$$\frac{\partial}{\partial x} = \gamma\left(\frac{\partial}{\partial x'} - \frac{v}{c^2}\frac{\partial}{\partial t'}\right), \tag{30}$$

Finally, one subtracts the relations in (26)

$$-\frac{2}{c}\frac{\partial}{\partial t} = A\left(\frac{\partial}{\partial x'} - \frac{1}{c}\frac{\partial}{\partial t'}\right) - A^{-1}\left(\frac{\partial}{\partial x'} + \frac{1}{c}\frac{\partial}{\partial t'}\right), \tag{31}$$

and uses (28) to obtain the Lorentz transformation for the time-derivative operator

$$\frac{\partial}{\partial t} = \gamma\left(\frac{\partial}{\partial t'} - v\frac{\partial}{\partial x'}\right). \tag{32}$$

Using (30) and (32) and following derivation III one gets the Lorentz transformations

$$x' = \gamma(x - vt), \quad t' = \gamma\left(t - \frac{vx}{c^2}\right), \tag{33}$$

3. Pedagogical comments on the four derivations

Some brief comments on the four derivations of the Lorentz transformations are in line.

- Derivation I is a simple, elegant and brief derivation of the Lorentz transformations [4–6]. This derivation is clearly based on the two postulates of special relativity and considerations of homogeneity and isotropy of the space. It is pertinent to note that there are many derivations of the Lorentz transformations that start by assuming the form invariance of the quantity $x^2 - c^2t^2$ [7–10]. A pedagogical advantage of considering the invariance of $x^2 - c^2t^2$ is that it can be justified from both a geometrical and a physical point of view. The instructor can invoke a geometrical argument by stating that the invariance of this quantity means the invariance of the Minkowski space-time interval: $S^2 = x^2 - c^2t^2 = x'^2 - c^2t'^2 = S'^2$. He can also use a physical argument by stating that the mentioned invariance means the invariance of the light front-wave equation: $0 = x^2 - c^2t^2 = x'^2 - c^2t'^2 = 0$.
- Derivation II was proposed by Parker and Schmieg several years ago [11]. It is also simple, elegant, brief and clearly based on the two postulates of special relativity and the homogeneity and isotropy of the space. The authors called the relations $x' - ct' = A(x - ct)$ and $x' + ct' = A^{-1}(x + ct)$ (with A and A^{-1} defined in (11)) the

diagonal form of the Lorentz transformations in the standard configuration. This derivation has the practical advantage that it involves only one effective constant to be found, namely, A in equation (11). In contrast, derivation I involves three constants to be determined (A, C and D in (3)). After introducing the Lorentz transformations following derivation I, the instructor can find useful to re-derive these transformations following derivation II.

- Derivation III is introduced here. It is similar to derivation I in the sense that both of them involve three constants to be determined. However, they differ in some respects. Derivation I starts with the algebraic expression (1) and derivation III with the differential expression (16). A practical advantage of derivation I with respect to derivation III is that the former is shorter than the latter. While derivation I involves algebraic computations, derivation III additionally involves differential calculations. For a historical introduction to special relativity, the instructor can find preferable to present first derivation III after introducing derivation I. The reason is quite simple: covariance of the homogeneous wave equation in inertial frames was first considered by Voigt [12] in 1887. However, Voigt derived space-time transformations different from those of Lorentz [13]. The invariance of the homogeneous wave equation was demonstrated by Poincaré [14] in 1905.
- Derivation IV is the same as I have recently presented [1]. Derivations II and IV are similar in most aspects. The basic difference between both derivations is the starting point, namely, equation (8) for the former derivation and equation (25) for the latter one. However, derivation II is shorter than derivation IV. Derivation IV is also simple and elegant. By considering the same historical aspects mentioned for derivation III, the instructor can prefer to first present derivation IV.

4. On the comments by Di Rocco

- In his comment, Di Rocco [2] suggests to replace the following statement appearing in my original paper [1]: 'By assuming linearity for the involved transformations of operators, we can write..' by the more appropriate statement: 'Considering the *necessary linearity* for the involved transformation of operators, we write..' I agree with the suggestion of Di Rocco. The operators are necessarily linear because of the homogeneity and isotropy of the space [4].
- Di Rocco claims that [2]: '... from the fact that transformations between pairs (x, t) and (x', t') *must be linear*, and with no other assumption we arrive to the LT' (italic emphasis mine). However, besides the linear form of the starting transformations, Di Rocco additionally *assumes* the form invariance of the homogeneous wave equation in his derivation of the Lorentz transformations. Furthermore, when introducing the inverse transformations by means of (35) Di Rocco makes the replacement $v \rightarrow -v$. Sardelis [15] has convincingly pointed out that such a replacement should not be seen as a consequence of the principle of relativity but as a consequence of *assuming* the isotropy of space.
- With respect to my original derivation of the Lorentz transformations [1], Di Rocco claims in his comment [2] that 'It must be clear that it is not necessary to demand the Galilean limit, because $\partial/\partial t' = v\partial/\partial x'$ every time $\partial/\partial t = 0$.' However, the claim of Di Rocco is equivalent to that implied by the Galilean transformation $\partial/\partial t = \partial/\partial t' - v\partial/\partial x'$. From this it follows that if $\partial/\partial t = 0$ then $\partial/\partial t' = v\partial/\partial x'$.

The fact that Di Rocco does not mention the Galilean limit, does not mean that such a limit is absent in his derivation. Moreover, Di Rocco starts with $x' = \mathrm{A}x + \mathrm{B}t$ and claims that for dimensional reasons the relative velocity is given by $v = -\mathrm{B}/\mathrm{A}$ and then $x' = \mathrm{A}(x - vt)$. But this argument is formally equivalent to that based on the Galilean transformation $x' = x - vt$, according to which the origin of the primed frame $x' = 0$ is the point $x = vt$ described in the unprimed frame. Using these conditions in $x' = \mathrm{A}x + \mathrm{B}t$ one gets $v = -\mathrm{B}/\mathrm{A}$ and hence $x' = \mathrm{A}(x - vt)$. In any case, from purely physical considerations the Galilean limit must be satisfied.

- As above mentioned, Di Rocco [2] uses the standard configuration and starts his derivation with the transformations $x' = \mathrm{A}x + \mathrm{B}t$ and $t' = \mathrm{C}x + \mathrm{D}t$ between the coordinates (x, t) and the coordinates (x', t'). Di Rocco observes that $\mathrm{B} = -v\mathrm{A}$ for dimensional considerations. He then writes the starting transformations as

$$x' = \mathrm{A}(x - vt), \quad t' = \mathrm{C}x + \mathrm{D}t, \tag{34}$$

and their associated inverse transformations as

$$x = \frac{\mathrm{D}x' + v\mathrm{A}t'}{A(\mathrm{D} + v\mathrm{C})}, \quad t' = \frac{-\mathrm{C}x' + \mathrm{A}t'}{A(\mathrm{D} + v\mathrm{C})} \tag{35}$$

By making use of (34) he derives the transformations between derivative operators: $\partial/\partial x = \mathrm{A}\partial/\partial x' + \mathrm{C}\partial/\partial t'$ and $\partial/\partial t = -v\mathrm{A}\partial/\partial x' + \mathrm{D}\partial/\partial t'$. Using these transformations and the homogeneous wave equation $\partial^2\Psi/\partial x^2 = (1/c^2)\partial^2\Psi/\partial t^2$ he obtains the equation

$$\begin{aligned}(\mathrm{A}^2 - \mathrm{A}^2v^2/c^2)\frac{\partial^2\Psi}{\partial x'^2} + 2(\mathrm{AC} + v\mathrm{AD}/c^2)\frac{\partial^2\Psi}{\partial x'\partial t'}\\ = (\mathrm{D}^2/c^2 - \mathrm{C}^2)\frac{\partial^2\Psi}{\partial t'^2},\end{aligned} \tag{36}$$

and then assumes the form invariance of the wave equation to find the relations $\mathrm{D} = \mathrm{A}$ and $\mathrm{C} = -v\mathrm{A}/c^2$. By substituting these values in equations (34) and (35) they take the form $x' = \mathrm{A}(x - vt)$, $t' = \mathrm{A}(t - vx/c^2)$, $x = (x' + vt')/[\mathrm{A}(1 - v^2/c^2)]$ and $t = (t' + vx'/c^2)/[\mathrm{A}(1 - v^2/c^2)]$. He then claims 'To keep the form between pairs x' and x as well as between t' and t, it is required that $\mathrm{A} = 1/[\mathrm{A}(1 - v^2/c^2)]$ so $\mathrm{A} = 1\big/\sqrt{1 - v^2/c^2} = \gamma$, and therefore $\mathrm{D} = \gamma$ and $\mathrm{C} = -v\gamma/c^2$.' Using these values for A, C and D the relations in (34) identify with the Lorentz transformations. This derivation of the Lorentz transformations suggested by Di Rocco is correct but criticizable in the following two pedagogical aspects.
- The inverse transformations in (35) *are not necessary* in the derivation of the Lorentz transformation suggested by Di Rocco. From (36) it is clear that the invariance of the wave equation requires that the constants A, D and C must satisfy the system of algebraic equations $\mathrm{A}^2 - \mathrm{A}^2v^2/c^2 = 1$, $\mathrm{AC} + v\mathrm{AD}/c^2 = 0$ and $\mathrm{D}^2/c^2 - \mathrm{C}^2 = 1/c^2$ (notice that this system is equivalent to that in (5)). From the first equation one directly obtains $\mathrm{A} = 1\big/\sqrt{1 - v^2/c^2} = \gamma$. The second equation reduces to the equation $\mathrm{C} + v\mathrm{D}/c^2 = 0$, which combines with the third equation to yield the values $\mathrm{D} = 1\big/\sqrt{1 - v^2/c^2} = \gamma$ and $\mathrm{C} = -v\big/[c^2\sqrt{1 - v^2/c^2}]$. Substituting these values in (34) one obtains directly the Lorentz transformations without the need to use the inverse transformations given in (35).
- The derivation of Di Rocco is to a certain extent a combination of derivations I and III. However, I find this combination little attractive from a pedagogical point of view. Let me explain. Di Rocco begins with the starting transformations

$x' = \mathrm{A}(x - vt)$ and $t' = \mathrm{C}x + \mathrm{D}t$ and calculates their associated transformations $\partial/\partial x = \mathrm{A}\partial/\partial x' + \mathrm{C}\partial/\partial t'$ and $\partial/\partial t = -v\mathrm{A}\partial/\partial x' + \mathrm{D}\partial/\partial t'$. His plan is to use the invariance of the wave equation to find the values of the constants A, C and D and then to substitute them in the starting transformations to obtain the Lorentz transformations. But if one begins with the transformations $x' = \mathrm{A}(x - vt)$ and $t' = \mathrm{C}x + \mathrm{D}t$, it turns out to be definitely simpler to use the invariance of the Minkowski space-time interval, $x'^2 - c^2t'^2 = x^2 - c^2t^2$, to obtain the Lorentz transformations thus avoiding the use of the homogeneous wave equation (see derivation I). Of course, this is not a serious objection to Di Rocco's derivation. It can be classified as a matter of taste.

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